

There is a type of substitution which, for lack of a better name, could be called

Brute Force Substitution

In Brute Force Substitution, you let u = a very complex part of the integrand (which may include a function composition, but usually not a product, quotient, sum or difference), then solve for the original variable, and proceed with the substitution and see where it takes you.

This type of substitution is similar to trigonometric substitution, except that the original variable is replaced with other types of functions.

Consider $\int \arcsin \sqrt{x} \, dx$.

In a regular substitution, you might set $u = \sqrt{x}$ since \sqrt{x} is the inner function

$$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$$

$$dx = 2\sqrt{x} \, du$$

$$\arcsin \sqrt{x} \, dx = (\arcsin \sqrt{x})(2\sqrt{x}) \, du = 2u \arcsin u \, du$$

$\int 2u \arcsin u \, du$ can then be evaluated by using integration by parts followed by a trigonometric substitution. (Try it.)

So, altogether, 3 different techniques were required.

In a brute force substitution, you might set $u = \arcsin \sqrt{x}$

$$x = (\sin u)^2$$

$$dx = 2 \sin u \cos u \, du$$

$$\arcsin \sqrt{x} \, dx = (\arcsin \sqrt{x})(2 \sin u \cos u) \, du = u \sin 2u \, du$$

$\int u \sin 2u \, du$ can then be evaluated by using only integration by parts. (Try it.)

So, altogether, only 2 different techniques were required.

Consider $\int \frac{1}{\sqrt{x} + \sqrt[6]{x}} \, dx$.

In a regular substitution, the obvious inside function is $\sqrt{x} + \sqrt[6]{x}$, but its derivative is rather ugly and hard to identify as a factor.

Consider the brute force substitution,

$$u = \sqrt[6]{x}$$

$$x = u^6$$

$$dx = 6u^5 \, du$$

$$\frac{1}{\sqrt{x} + \sqrt[6]{x}} \, dx = \frac{1}{\sqrt{u^6} + \sqrt[6]{u^6}} 6u^5 \, du = \frac{6u^5}{u^3 + u} \, du = \frac{6u^4}{u^2 + 1} \, du$$

$\int \frac{6u^4}{u^2 + 1} \, du$ can then be evaluated by using polynomial long division. (Try it.)

So, when using substitution on a complex integrand, try being as aggressive as possible in your choice of the substitution.